Department of PHYSICS

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INSIDE:

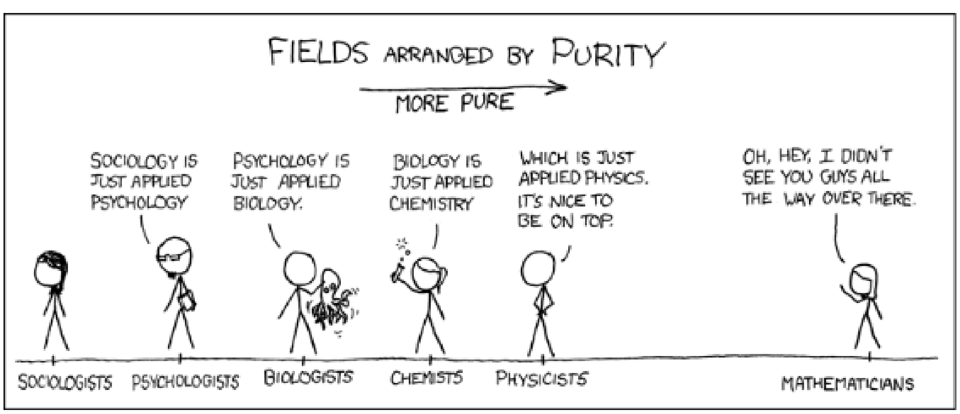
UNPHYSICAL

Physics and Interdisciplinary Research

- Guru Vamsi & Arkya Chatterjee, *4th year EP*

Physicists today are regularly moving out of their traditional comfort zone. They're redefining the scope of the subject and tackling problems in areas as diverse as biology, computer science, and information theory. This article aims to be an exposition of interdisciplinary work that derives heavily from physical laws and principles. In the process, we hope to convince the reader that getting a degree in physics goes beyond the conventional; it can be a foray into fascinating topics like population dynamics, evolution, and cryptography.

Physics is inherently concerned with reducing large complicated problems to its bare essentials. As a common joke goes, the life of a theoretical physicist involves solving the simple harmonic oscillator in increasing degrees of abstraction. Given any physical problem, they look to strip away the cosmetic details, focusing on what they believe to be the core principles. Biology, on the other hand, is at the other end of the spectrum, with its complicated genetic networks, evolutionary pathways, and protein structures. This leads to a rather oxymoronic phrase when one talks about 'biological physics'! How can two fields that are philosophically so different possibly be united?

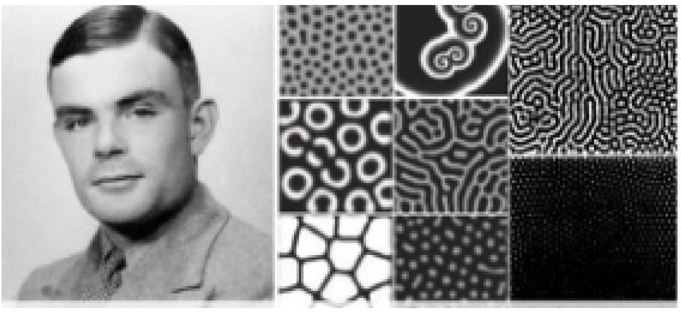


Let's take an example to clarify this: patterns are pretty common both in biology and physics. Ferromagnetic materials (such as iron) show the interesting property of retaining their magnetisation even after the magnetising agent (for example, a current loop) is removed. This is how permanent magnets are created. This phenomenon is described in terms of patterns ("long-range order") created out of the alignment of electronic spins. In 1924, Wilhelm Lenz and Ernst Ising developed the Ising Model to explain ferromagnetism, or more particularly, the phase transition from ***ferro***magnetic to ***para***magnetic behaviour. Quite interestingly, biological physicists have recently started applying the Ising model as a framework for phase transitions in cancerous tissues[1]. There have also been attempts at Ising model-inspired modelling of gene regulatory networks[2] that aim to describe phase transitions in multicellular environments. What these two examples have

[1] ***https:/arxiv.org/abs/1010.6284***
[2] ***https:/www.ncbi.nlm.nih.gov/pmc/articles/PMC4938077/***

in common is the property of of self-organization. Based on certain parameters, systems like these can either show an ordered behaviour (with interesting patterns), or a disordered, random one. It's mind-blowing how a model for magnetic materials can be applied successfully to study biological systems that have seemingly nothing to do with physics.

If the reader is now convinced that the term 'biological physics' is not so paradoxical, we have more examples that might be interesting. Morphogenesis is the biological process that causes an organism to develop its shape.



Alan Turing, in his seminal 1952 paper[3], suggested that differential equations describing 'reaction-diffusion systems' - physical systems where chemical reactions and concentration gradients cause spatial and temporal variations - would be able to account for the phenomenon of morphogenesis. In this way, he was able to explain how patterns like the ones on a leopard's body, might arise depending on different boundary conditions applied on the differential equations.

Morphogenesis has also been studied by soft condensed matter physicists over the last few decades. For example, Prof. Amitabha Nandi from the Department of Physics, IIT Bombay, has worked on the development of the Drosophila pupal wing[4], with the aim of explaining the shape acquired by the tissue based on epithelial stress and cellular dynamics. This is yet another example of physical laws bringing about ordering in a biological setting. 'Order' is inherently tied to the concept of entropy - the degree of disorder in any system. The second law of thermodynamics, that deals with entropy, manifests itself beautifully in the living world, where we find biological organisms seemingly creating order out of chaos. Obviously, this decrease in randomness is at the expense of increasing the entropy of the universe. With that broad outlook, physicists like Nigel Goldenfeld and Jeremy England have been trying to tackle the physics behind life and the evolution of complex biological systems. England has proposed a thermodynamic framework behind the origin of life. Computer simulations[5] of chemical reaction networks have shown the emergence of a stable ordered state in a highly non-equilibrium environment as a consequence of a small set of thermodynamic principles, lending some justification to the theory. Goldenfeld, on the other hand, has been pursuing a theory of evolution for the pre-biological era of life, i.e. before there were genes and species.

The way evolution has taken place since the last universal common ancestor[6], living around 3.8 billion years ago, is via gene transfer from parent to offspring. But such a transfer will be slow and cannot account for the fact that life went from a few simple molecules to the huge complexity of the first cell in a few million years. Instead, 'horizontal' gene transfer, i.e. exchange between all members in a collective state, could potentially explain this discrepancy. Fortunately enough, this kind of collective behaviour is quite common in condensed matter physics, and it might be possible to end up with a full-fledged physical theory describing the development of complexity at the dawn of life.

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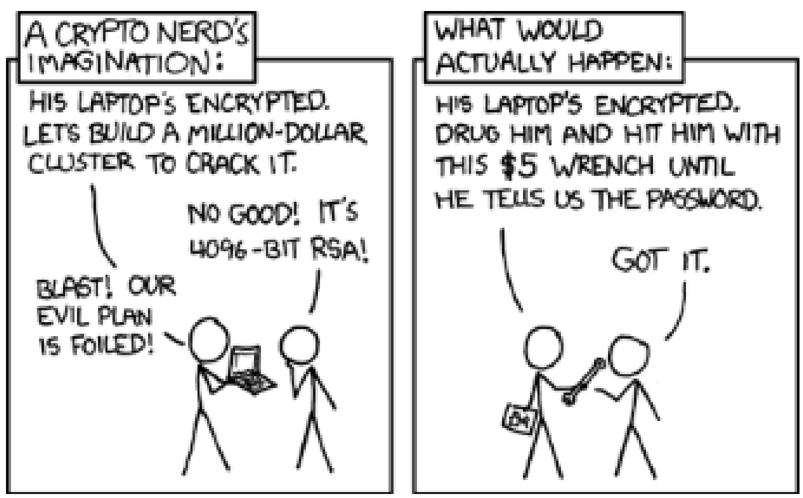
Contrary to Biology, Computer Science is much closer to how a Physicist thinks. Computer Scientists are obsessed with abstraction and quite often it becomes a necessary perspective to grant a layman access to technology. In this section, we begin by giving a flavour of modern cryptography to the reader and then discuss how physics has paved the way for a completely different approach.

[3] *http:/rstb.royalsocietypublishing.org/content/237/641/37.short*
[4] *https:/elifesciences.org/articles/07090*
[5] *https:/www.pnas.org/content/114/29/7565*
[6] *https:/en.wikipedia.org/wiki/Last_universal_common_ancestor*

After the second world war, cryptography quickly rose to popularity for commercial purposes. Partly driven by industrialization and companies fighting to keep each other's trade secrets, this quest to encipher information to make it secure became vital in understanding what is feasible within fundamental limitations of what one can do.

One important figure that laid the foundations of information theory is Claude Shannon. He was able to prove that the amount of randomness needed to completely hide a message is greater than or equal to the length of the message. Any less would leak information. However, this is never done in practice because it's absurd to have a key as long as the message, effectively doubling the amount of information transferred just to keep it secure! In the real world, people assume a computational limit on adversaries, this simplifies things and allows for



much more flexibility. Things like Public Key Encryption (used when ordering from online retailers like Amazon) or even a simple random number generator are secure only under the assumption that the adversary does not have an infinite amount of time to crack the system.

Over the past few decades, cryptography has entered a new phase where it's no longer about communicating securely. The focus has shifted to things like computing securely, digital cash, obfuscating programs, private information retrieval, to name a few.

Take the example of secure Multi-Party Computation (MPC). The goal of MPC is to allow multiple parties to evaluate a function which takes inputs from all parties. The tricky part is - no party should learn anything other than the final answer. Let's flesh this out with a story - say two millionaires had lunch together. They decide that that the richer one pays the bill, but neither of them wants to reveal their actual worth to the other, nor do they trust a third party to look at their bank accounts and decide who's richer. Solving such seemingly impossible tasks is the goal of Multi-Party Computation. Doing this with honest parties who follow the protocol is in itself a hard task. In real life we seldom have situations where everyone is honest, which would mean we have to account for situations where a party tries to cheat by giving false inputs and tries to gain information about the other inputs (often modeled under the name of Active Adversaries).

A cryptographer loves problems that are hard to solve, not because he can solve them but because he uses their 'hardness' as a means to an end. In this view, ideas from topics in Physics like Quantum information and Non-Linear Dynamics can often be seen in close contact with cryptography. Take Non-Linear Dynamics for starters, chaos is a phenomenon where a system evolves haphazardly but deterministically. The large amount of entropy involved in the process and the non periodicity of the system allows one to extract pseudo-randomness from it; thus building a pseudo-random number generator!

Generating truly random numbers has always been a challenge, people settle for things that are very close to one. Quantum states can be used to generate truly random numbers. A feat previously unheard of using any deterministic algorithm run on a classical computer (obviously!).

This is where Quantum Information comes in. Quantum Key distribution is another popular scheme for exchanging keys that can later be used for communication. Not just that, as many of the readers may already know, Quantum computers can potentially be used to break a few classical encryption schemes based on factorization and the discrete log assumption as well[7]. So great is the impetus of this radical idea that cryptographers have now begun to study an entirely new division called post-quantum cryptography in an attempt to produce schemes secure against Quantum Computers! The frontrunners in post-quantum cryptography are lattice based schemes, where the hardness in solving certain lattice problems are used to build cryptographic protocols.

Finally, let's look at one of the most fundamental tools in modern cryptography - secret sharing. Secret sharing is a primitive where a dealer splits a message into n pieces (one piece given to each party) (say) and with any t(>n) pieces one can recover the message (effectively any t parties can recover the message). It finds applications in Missile Launch code encryption, Numbered bank accounts and is an important tool in Multi Party Computation. Imagine you're the president of a country who nobody trusts with the nukes[8]. The shares can be distributed in such a way that the president gets veto power but needs the consent of say ⅗ security generals in order to launch a missile. This prevents complete autonomy and eliminates the need for parties to trust one another. Although this doesn't seem too hard at first glance, the difficulty arises when one requires that any collection of pieces less than t, in number should reveal NOTHING about the message. However, things get even trickier when people deviate from the assigned protocol and start announcing wrong shares. Attacks could be such that only the dishonest party recovers the actual message and the honest parties recover a wrong message. In order to tackle this type of adversaries one of the authors has worked with Prof. Punit Parmananda from the physics department to prevent attacks by such dishonest parties using ideas from nonlinear dynamics (in particular chaos and sensitive dependence on initial conditions.)[9]

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The possibilities are immense when it comes to interdisciplinary research. In this article, we have only explored a small number of examples from two broad categories that present themselves at the synaptic end of the subject. There are tons of open problems in biology and computer science that are routinely being targeted with this interdisciplinary ideology.And we believe that to an inquisitive and insightful Physicist, many more await.

[7] The discrete log assumption - The logarithm of a number is defined as the exponent to which a number(base) must be raised to in order to the produce the former. Analogously, the discrete logarithm is defined for elements in a Group(a set which allows for operations on it's elements). Calculating the discrete log is believed to be hard in certain groups. Hence known as the Discrete Log "Assumption".
[8] Legal issues prevent us from taking names
[9] http:/bit.ly/2G9Ob58 - A poster on Secret Sharing using Chaos

From ZURICH *to* MUMBAI

Physics at IIT Bombay

- by Fabian Binkert,
Exchange Student, ETH Zurich, Switzerland

Many of my friends back home asked me: "Why India?" with a mixture of surprise and disbelief on their faces when I told them I will go for an exchange semester to India. Back then, I was not really able to give them a convincing answer as to "why". I was not even sure myself why I picked IIT Bombay as my exchange destination.

I had been studying physics for three years at ETH Zurich in Switzerland when I completed my bachelor's degree last summer. At that time I knew I wanted to experience something new. Don't get me wrong, I love studying physics at ETH. The professors who have become my mentors over the years are excellent tutors. The peers, with whom I had spent many summer nights at the library, have become great friends and the research facilities are among the best in the world. Yet, the place has become very familiar, maybe too familiar. Therefore, when I had the chance to spend the first semester of my master's degree abroad, I went for it. I could have chosen many different places but I chose to come to India. I did not know much about India and I chose it simply based on my gut feeling and the hope that it would be a different and refreshing experience. After that, the decision to apply for a place at IIT Bombay was not difficult for me. Not only because IITB has a very good reputation but also because the IIT system produces so many talents who are able to work anywhere in the world. Coincidentally, one IIT graduate ended up being my teaching assistant at ETH for a project on solid-state lasers which I did in the quantum optics laboratory. Before my departure to India, I pestered him with questions about his studying experience until I was confident to have made the right decision. That confidence almost immediately vanished after my arrival here in India back in December. I felt so lost among the humongous amount of people everywhere. The never ending traffic and the constant honking on the streets of Mumbai put me under constant stress which was really hard to bear. Fortunately, the IITB campus is a green and calm oasis within the nerve racking hustle and bustle of the city. It allowed me to expose myself in small doses to the Mumbai way of life and get accustomed slowly.

Now, I have been here for three months and my perspective has changed completely. Everything seems so normal now and I have learned to love Mumbai despite, or perhaps due to all the differences I experience every day compared to my daily life in Zurich. A major catalyst to that transition were the students of IITB who are always ready to give a helping hand. I was especially thankful for that during the first days of the semester. I had no clue how the academic system worked here and I realized quickly that everything is a bit less rigorously organized than back home. So I was more than happy to be able to rely on the advice of my fellow students who helped me with all of my questions. After the first week I had no longer any troubles with finding all the classrooms at the right time. However, what caused inconvenience for me a bit longer, was the fact that I have never had any quizzes and mid-sems during the semester. That meant, I had to adapt my study habits pretty quickly to the IIT system. At ETH we do not have any graded tests during the semester at all. There is usually only one exam for each course, not right at the end of the semester but just before the start of the new semester. That means there is always a two months gap between the end of the courses and the exams. The good thing about that gap is that there is always plenty of time to take another look at a topic after the semester, in case I did not fully understand it right away. Also, during the semester I was able to put my focus on understanding the content and hand in qualitatively good assignments without worrying too much about exams. Exam preparation is what I always did after the semester. The downside of that is that usually the entire winter and summer breaks are spent at the library.

Here at IITB, we have to prepare for either quizzes, mid-sems or end-sems every couple of weeks. On top of that, we have to keep up with the assignments at the same time. The IITB system is great for me because it effectively keeps me from procrastinating and it forces me to understand everything right away. On the other hand, I have realized that I focus on solving exercises most efficiently rather than trying to understand the physics behind it. All in all I feel like the IITB system gives me a bit less personal freedom. However, through the frequent exams I get an excellent feedback about my progress during the semester.

A difference inside the classroom is the fact that the number of students in each of my courses here at IITB is much lower than what I am used to at ETH. During my bachelor's courses, the number of students in a lecture ranged from about a 100 to 300, depending on the course. This large number of students makes student life at ETH very anonymous. Whereas, the lectures in the department of physics at IITB are very intimate. In my courses, there are only about 15 to 30 students in each class. Also, I think more than half of my professors remember me by name. Well, this is not a very great achievement because as the only exchange student in a small class I stand out a lot. But still, it is a very nice experience. Generally, there is much more interaction between students and professors in the lectures than what I was used to at ETH which I like.

Even though these points are characteristics of IITB and ETH, I think studying physics at either of these institutions is not much different. The quality of teaching is high at both universities and as far as I know, the laws of physics are the same in India or Switzerland. I can also confirm, proofing tensor identities is a pain no matter where you do it.

What actually makes the experience here so refreshing for me is the whole life outside the classrooms. Among that, the people who make me feel more than welcome and the kindness that I encounter everyday. I am looking forward to report to my friends that these wonderful experiences are the reason why I enjoy my stay at IITB so much and I would choose it as an exchange destination anytime again.

What do BLACK HOLES *and* SUPERCONDUCTORS *have in common?*

What do BLACK-HOLES *and* SUPERCONDUCTORS *have in common?*

- Vaibhav Sharma, *MSc 2018*

Black holes and superconductors are not concepts one usually encounters in the same place. The former are described by Einstein's theory of general relativity while the latter is the poster child of macroscopic quantum mechanical effects. And as is known, these two theories do not particularly get along well. However, recent research suggests that these two disparate phenomenon can be connected by a strange duality.

Duality refers to an object inherently having distinct, discernible features of two very separate perspectives. To the layman, the philosophical duality of the glass simultaneously being half empty and half full is a good example. To the phycisist, the wave-particle duality of light and matter comes to mind. Oftentimes, two different views can be equally valid to describe an object or an observation. And in this article, we explore the possibility of such a duality existing between black holes and superconductors.

Before we embark onto this adventure, let's take a step back and review what we know about these two topics. Superconductivity is a 'phase' where a material can support persistent currents. The Bardeen-Cooper-Schriefer (BCS) theory of 1957 was incredibly successful in describing conventional superconductors, which were experimentally observed as early as 1911. The theory propounded an effective attraction between electrons at low temperatures with the help of crystal vibrations ('phonons') to pair them up (into Cooper pairs) to explain the phenomenon. The idea was that while a single electron is a Fermion which keeps its distance from other electrons because of the Pauli Exclusion Principle, a pair acts like a Boson, inheriting the Bosonic tendency to come together. These Cooper pairs accumulate to form a condensate, supporting persistent currents and thus causing superconductivity. There is a critical temperature for each material which can support the phenomena, above which the superconducting condensate is not formed. And to understand simply, this critical temperature is controlled by the phonon induced coupling.

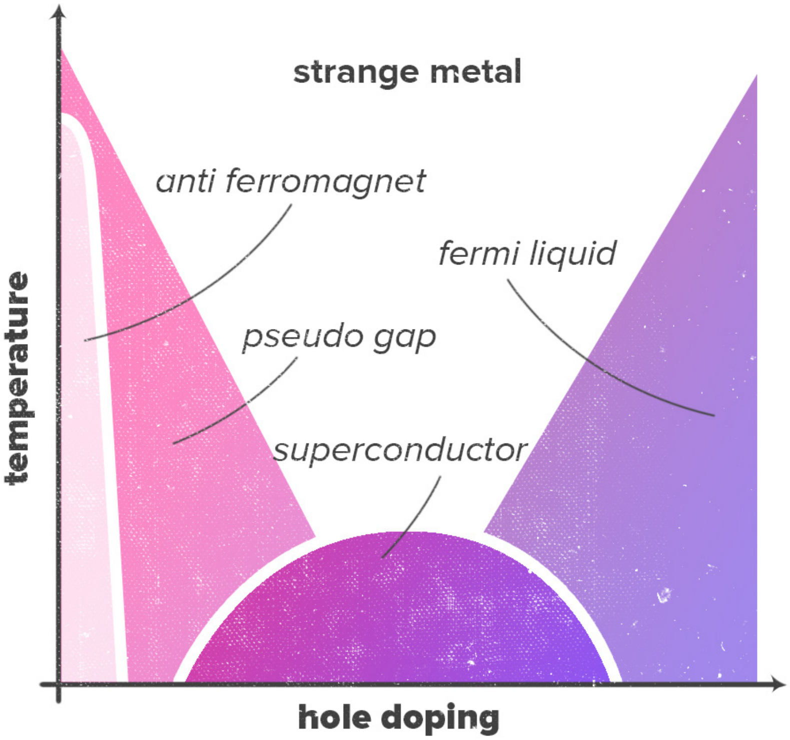
Black holes, on the other hand, are solutions to Einstein's general relativity equations. What most people do know about this astronomical entity is that it has a well-defined boundary called the 'event horizon' beyond which even light cannot escape. We try to understand them better through the famous Hawking theory of black-hole dynamics. Black holes have an analogue of the Second Law of Thermodynamics where the area of the black hole's event horizon is said to carry the entropy. This ensures that whenever something falls inside, the black hole along with its event horizon becomes bigger, thus preserving the law which states that, in totality, entropy must always increase. In fact, the area of the black hole gives the amount of entropy in it! They also have their own temperature, thus making them radiate like ordinary black bodies. This Hawking radiation defines the Hawking black-hole temperature to be -

$$\mathcal{T} = \hbar^*\kappa$$

the surface gravity, depends on the mass of the black-hole, and can be intuitively understood as the acceleration needed to keep something at the horizon without getting sucked in.

Now that we can imagine how different and orders of magnitude apart these two objects are, it is interesting to see what prompted physicists to try and connect them. The BCS theory, though successful in conventional superconductors, fails to explain superconductivity at high critical temperatures (around 80-100 K) in a certain class of doped materials called cuprates. We don't exactly know why or how superconductivity works in these materials, and more importantly we don't understand what sets the high critical temperature of the condensate in such cases.

On the next page is a picture of the phase diagram of cuprates. On the extreme left, when doping is low and the temperature is cold, we have an antiferromagnetic insulator while on the right, the material behaves as a metal (or fancily called, Fermi liquid or Fermi electron gas). It helps to understand



this diagram by picturing water and its phases. Below 273K(0C) under standard conditions we have an ordered solid, ice while above 373 K (100C) we get water vapour, which is a gas. Similarly, the insulator here is an ordered solid while the metal is like an electron gas. And just like we have water in the middle, somewhere between solid-like order and gas-like disorder, there is a quantum liquid or strange metal in the diagram.

Quantum liquid is a disordered state. It has no long range order and yet is not an electron gas. It also has high entropy and exists at high temperatures when doping is intermediate. Once in this region of the phase diagram, we can carefully lower the temperature to enter the superconducting dome. Now how do we build superconducting order out of this quantum liquid state? And what really sets the critical temperature of these new formed superconductors? Such fundamental questions have so far eluded an explanation. And a fresh look at them is what the titular duality promises.

With the basic concepts in hand, we can go ahead and see what links them. At this point, we need to know that the black-hole under scrutiny is a rather unusual one.

Imagine this black-hole to be in a box under thermal equilibrium, that is, its temperature is equal to the temperature of the box and therefore, it doesn't radiate any energy. Now, apply some positive charge on the black hole - this creates an electric field around it. This electric field that exists in free space causes the interesting phenomenon of Schwinger pair production, wherein a pair of oppositely charged particle-antiparticle duo are created. To see that this is energetically feasible, one should note that this pair will have its own new electric field, opposite in direction to the original field, effectively reducing the net electric field between them. The missing energy can then be equated to the mass energy of the particles that were formed. In this way, near the horizon, such pairs can form spontaneously, by themselves, without any external assistance.

The negative charge in this pair will be attracted by the positively charged black-hole by both the electromagnetic and gravitational forces. But the dynamics of the positive charge of the pair is interesting - while gravitationally, it feels attraction from the massive object, there is also an electrostatic repulsive force owing to its positive charge.

As we saw earlier, the size of the black-hole decides its surface gravity which in turn sets its Hawking temperature. If the black hole has a higher temperature (higher surface gravity), gravity wins and the positive charge falls into the black hole along with the negative charge and the black-hole becomes bigger. However when the temperature is low (low surface gravity), electromagnetic force can win and the positive charge can fly off, until it reaches a distance where gravity and electromagnetic forces balance out. This would create a condensate of positive charge around the black hole. We can see that there is a critical temperature for this black-hole below which a transition to the condensate phase becomes possible.

Now, where have we seen the phenomenon of formation of condensates as temperature is lowered? You're right, in superconductors!

Both black-holes and quantum liquids have high entropy. The usual difficulty that arises with condensed matter systems like superconductors is the need for complicated formulations such as many-body quantum theory to successfully solve them. But this crucial observation gives condensed matter physicists a new tool to understand this problem. One can describe black-hole dynamics by general relativity and then find solutions that shed light on what controls the critical temperature. Then the duality can be exploited to gain insights into what happens with high-temperature superconductors!

In this way, condensed matter physicists can now see a novel perspective towards solving some of their intractable problems -- in the seemingly unrelated topic of General Relativity.

IIT *is not only about* ENGINEERING

The road to M.Sc. and beyond

- Jainam Khera, *2nd Year MSc*

"Beta, aap kahan padhte ho?"
"IIT Bombay."
"Accha, toh aapne JEE diya hoga na? Kaunsi engineering kar rahe ho aap?"

This is Jainam Khara, 23 and a student currently enrolled in the first year of M.Sc. Physics programme here at IIT Bombay. To the readers who are surprised, "Jee haan! IIT me M.Sc. bhi hota hai." On the occasion of the Diamond Jubilee of IIT Bombay, I wish to make people aware of the M.Sc. Programme here.

I would first like to emphasise upon the perks of studying at IIT Bombay and then I shall take it down to the Physics Department and then to the M.Sc. Course. One just cannot deny that IIT Bombay has a brand name, it gives you some credibility and it fetches you the necessary platform for your career. Apart from that, in my personal opinion, IIT Bombay creates experiences that students don't forget. If you are passionate about your interests and hoping for a peer group that consists of equally passionate individuals, I have certainly found this to be the right place.

Something few people know about is the number of extracurricular activities that the students are encouraged to participate in. To remember, graduate studies is not only about academics; it is also about life beyond academics - and this aspect is not lost among the many achievements that the Institute makes to propel research in India forward.

In the Physics department, research is conducted in several areas that are pushing the frontiers of our understanding of space and time, and of matter and energy in all its forms. The major attraction of the department is its distinguished faculty, some of the finest individuals across the country's academic landscape. Most of the professors, being deeply involved in the research within their subjects, make the best teachers and mentors for aspiring physicists. What's more, they often bring their expertise and enthusiasm to the classrooms, an attitude I have come to appreciate very much. The research topics vary of course, and span an astonishing breadth - High Energy, Condensed Matter, Soft Matter, Biophysics, Non-Linear Dynamics, Optics, Photonics, Astronomy, Cosmology and Gravitational Physics and many other interdisciplinary areas!

Every year around 12,000 students apply for IIT JAM (the count is however increasing by the years) and usually the ones placed among the top one percent get admission to IIT Bombay. This strict screening criteria has an upside - you get to study with some of the highly motivated graduate students in the country - striving together to create an impact. The curriculum, I believe, is quite rigorous. Each semester runs for about four months and in these four months you study 4 theory and 2 lab courses. With the continuous assessment system, you're usually busy through the semester with quizzes/projects, a mid semester and an end semester for each subject. Students are required to do a project under a professor here in their second year, but sometimes, highly enthusiastic ones may even approach their guide in their first year. I think everybody wants to run and run fast. There is also an internship cell which gives the first-year students several opportunities to pursue a summer research project abroad. Such a project could be of immense importance as it holds answers to questions about future career options. Besides providing a novel experience of another culture, students also begin diversify their CV and build research networks beyond their home institution.

What is beyond M.Sc.?

After an M.Sc. in Physics is that, there are no 'real' jobs for us. However, one always has the option to go for non-core jobs (in a non-scientific or engineering sector) or pursue teaching as a profession. What I see is that most of the students go for a PhD, and then maybe even further, a Post-Doc. With a vibrant and active Ph.D. program, in which about 100 research students are currently enrolled, IIT Bombay Physics thrives to be one of the leading research institutes across India. The way to PhD here, after an MSc is easier. A national level entrance like Gate or CSIR's score always helps, and apart from that, the institute takes its own written exam followed by an interview. Like most other places, the program leading to the Ph.D. degree involves a course credit requirement and a research project leading to a thesis submission.

I believe that the quality and volume of research from India is on the upswing. Thanks to the research teams working across the institutes of national importance across India, it is truly an inspiring time to be part of this community. So, that's all I wanted to convey to all those graduates who are not sure how to go about their passion in science after undergraduate studies. If you have any more queries about the Department of Physics here at IIT Bombay, I'd be happy to talk to you.

Stay motivated!

TENSORS

beyond symbolic manipulations

- Reebhu Bhattacharyya, *5th year MSc Maths*
March 7, 2018

Tensors are ubiquitous in physics, they crop up almost everywhere, well everywhere if you regard vectors to be 1-tensors but more on that later!

But if you ask a physics undergraduate to define what a tensor is, they are likely to describe tensors as " a collection of quantities that transform in a certain kind of way under transformation of coordinates". This is neither a precise defition nor easy to generalise, especially in the context of general relativity.

In this article, we hope to convey the beauty of the coordinate free approach to tensors and their existence as a separate entity, not just a particular representation in some choice of coordinate system, from which we can switch back and forth as and when needed.
First, we aim to describe some lesser emphasized facts about dual spaces, and sketch an important construction in mathematics, the tensor product.

1. Dual Space

Let V denote a finite dimensional vector space over a field F (for those unaware of what a field is, feel free to replace F with R or C, it really does not matter), say of dimension n. For any vector space W, we denote the set of linear transformations between V and W by $\mathcal{L}$(V, W). Note that F is a vector space over itself, we call $\mathcal{L}$(V, F) the set of linear functionals on V and denote it by V*. One easily checks that V* has the structure of a F-vector space itself and we call it the dual space of V.

Physicists encounter the dual space in a variety of situations, for example the bra and ket notation(-due to Dirac) in quantum mechanics or the notion of covariant and contravariant vectors in special relativity.

We will have more to say on this soon.

We fix a basis $\mathcal{B} = \{e_1, \cdots, e_n\}$ of V. We define the the elements

$$\omega^i \in \mathbb{V}^* \text{ for } 1 \leq i \leq n$$

by defining their action on basis elements as follows:

$$\omega^i(e_j) = \delta^i_j \quad 1 \leq i, j \leq n$$

Then any arbitrary element $\alpha \in V^*$
can be written as

$$\alpha = \sum_{i=1}^{n} \alpha(e_i)\omega^i$$

or more familiarly in Einstein notation as

$$\alpha = \alpha_i \omega^i$$

where $\alpha_i = \alpha(e_i) \in F$
We make two remarks.

Firstly, note that although physicists generally are inclined to assign scalar quantities (whatever they might mean) to expressions involving contraction of (all) indices, in this case it is not so; the notation although suggestive only indicates to take the required vector space addition of some multiples of ω_i's, giving a linear functional α which's a vector in V*.

Secondly, the above expression indeed gives back α since for any $x = x^k e_k \in$ V, on one hand

$$\alpha_i \omega^i(x^k e_k) = \alpha_i x^k \omega^i(e_k) = \alpha_i x^k \delta^i{}_k = \alpha_k x^k$$

whereas on the other,

$$\alpha(x) = \alpha(x^k e_k) = x^k \alpha(e_k) = x^k \alpha_k$$

Thus the ω^i's are like coordinate functionals, giving us the coordinates of any vector with respect to the basis $\mathcal{B}$. From the above, we see that they span V* and can be shown to be lienarly independent. Hence, $\mathcal{B}^* = \{\omega^i : 1 \leq i \leq n\}$ is a basis for V*, called the basis dual to $\mathcal{B}$. This also gives us a vector space isomorphism between V and V* via the map $\varphi : V \twoheadrightarrow V^*$ given by $\varphi(e_i) = \omega^i$.

We pause to remark that the above isomorphism is not natural in the following sense: it involves a choice of basis, choosing a different basis for V will give a different isomorphism.

However, we have a bijective linear map $\psi : V \twoheadrightarrow V^{**}$ defined by $\psi(v) = \psi_v \in V^{**}$ where $\psi_v(\omega) = \omega(v)$ for all $\omega \in V^*$. This map defines a canonical isomorphism $V \cong V^{**}$.

2. Tensor Product

We first define two general mathematical constructions:

Quotient Vector Space

Let V be a vector space and $W \subset V$ be a subspace. For any element $v \in V$, define $[v] := v + W = \{v + w : w \in W\}$; it is an affine subspace isomorphic to W containing v, in other words W translated by v. Then, the quotient space V/W is defined to be

$$V/W = \{[v] : v \in V\}$$

Another way to see it is as follows: define an equivalence relation $\sim$ on V by $v_1 \sim v_2$ iff $v_1 - v_2 \in W$. In this case $[v]$ is the equivalence class of v under this relation and the quotient space is the collection of all these equivalence classes.

Free Vector Space

Let S be any set. We define the free vector space on S over the field F denoted by V_S as follows: let $F(S, F)$ be the set of all (set-theoretic) functions from S to F. This is naturally a vector space over F. Now define

$$V_S := \{f \in F(S, F) : f(s)=0$$

for all but finitely many $s \in S\}$

Note that V_S is a subspace of $F(S, F)$. Also, given an element $s \in S$, we can identify it with the function $f_s \in V_S$, $f_s : S \twoheadrightarrow F$ given by $f_s(t) = 0$ if $t \neq s$ and $f_s(s) = 1$. Thus, we have a map $\iota : S \twoheadrightarrow V_S$, $\iota(s) = f_s$. One can check that $\{f_s : s \in S\}$ is a basis for V_S.

One can also characterize the free vector space on S via its universal property, given any vector space W and a set map $\eta : S \twoheadrightarrow W$, there exists a unique linear map $\tilde{\eta} : V \twoheadrightarrow W$ such that $\tilde{\eta} \circ \iota = \eta$.

At last we are in a position to define the tensor product. So, let A and B be two vector spaces over F, not necessarily finite dimensional. Let $S = A \times B$ be their Cartesian product.

Let V_S denote the free vector space on A×B. Now consider the following subsets of V_S

$$\mathcal{T}_1 := \{f_{(a+a',b)} - f_{(a,b)} - f_{(a',b)} : a, a' \in A, b \in B\}$$
$$\mathcal{T}_2 := \{f_{(a,b+b')} - f_{(a,b)} - f_{(a,b')} : a \in A, b, b' \in B\}$$
$$\mathcal{T}_3 := \{f_{(\alpha a,b)} - \alpha f_{(a,b)} : a \in A, b \in B, \alpha \in F\}$$
$$\mathcal{T}_4 := \{f_{(a,\alpha b)} - \alpha f_{(a,b)} : a \in A, b \in B, \alpha \in F\}$$

Let $\mathcal{T} = \mathrm{span}\{\mathcal{T}_1 \cup \mathcal{T}_2 \cup \mathcal{T}_3 \cup \mathcal{T}_4\}$. Then, we define the tensor product $A \otimes B$ to be the quotient space

$$A \otimes B = V_S / \mathcal{T}$$

We denote the equivalence class of $f_{(a,b)}$, by

$$a \otimes b := [f_{(a,b)}]$$

Quotienting out by the $\mathcal{T}_i$'s correspond to the following:

$$(a + a') \otimes b = a \otimes b + a' \otimes b$$
$$a \otimes (b + b') = a \otimes b + a \otimes b'$$
$$(\alpha a) \otimes b = a \otimes (\alpha b) = \alpha(a \otimes b)$$

Here is another way to view the tensor product of V and W. For a vector space A, by Bil(V, W; A), we will mean the set of bilinear maps $\varphi : V \times W \twoheadrightarrow A$, that is φ satisifies for $\alpha, \beta \in F$

$$\varphi(\alpha v_1 + \beta v_2, w) = \alpha\varphi(v_1, w) + \beta\varphi(v_2, w)$$
$$\varphi(v, \alpha w_1 + \beta w_2) = \alpha\varphi(v, w_1) + \beta\varphi(v, w_2)$$

Then, the universal property of tensor products is that given any $\varphi \in \mathrm{Bil}(V, W; A)$, $\exists$ a unique linear map $\Phi \in L(V \otimes W, A)$ such that $\Phi(v \otimes w) = \varphi(v, w)$ $\forall$ $v \in V, w \in W$.

More generally, the notion of bilinear maps extends to the notion of multinear maps $\mathrm{Mult}(V_1, \cdots, V_n; A)$ involving maps $\psi : V_1 \times \cdots \times V_n \twoheadrightarrow A$ that are linear in each variable separately. Then the universal property of tensor products states that given $\psi \in \mathrm{Mult}(V_1, \cdots, V_n; A)$, $\exists$ unique linear map $\Psi : V_1 \otimes \cdots \otimes V_n \twoheadrightarrow A$ satisfying

$$\psi(v_1, \cdots, v_n) = \Psi(v_1 \otimes \cdots \otimes v_n)$$

3. Covariant & Contravariant Tensors

Now, we are prepared to define what we mean by covariant and contravariant tensors. While we can more generally define A-valued tensors for any vector space A, for notational simplicity and since the general case is not needed here, we will stick to F valued tensors.

Fix a finite dimensional vector space V, say of dimension n.
For us, the set of natural numbers N = {0, 1, · · · }. Fix r, s $\in$ N. We will define what we mean by (r, s) tensors, also called r-contravariant s-covariant tensors.

Define the vector space of (r, s) tensors on V,

$$\mathcal{T}_s^r(V) := \text{Mult}(V^*, \cdots, V^*, V, \cdots, V; F)$$

where V* is repeated *r* times, and V *s* times.

Now we use the universal property of tensor products to get the following bijective correspondence:

$$\mathcal{T}_s^r(V) := \text{Mult}(V^*, \cdots, V^*, V, \cdots, V; F) \cong \mathcal{L}(\otimes^r V^* \otimes^s V, F) = (\otimes^r V^* \otimes^s V)^* \cong \otimes^r V^{**} \otimes^s V$$

Identifying V** with V, we get

$$\mathcal{T}_s^r(V) := \otimes^r V \otimes^s V^*$$

where $\otimes^r V \otimes^s V^* = V \otimes \cdots \otimes V \otimes V^* \otimes \cdots \otimes V^*$

with V repeated *r* times and V* repeated *s* times.

Now, it's time to make a few observations and fix more (hopefully sensible) notations. Firstly, we see $T_0^1(V) \cong V$ and $T_0^1(V) \cong V^*$.
By convention we fix $T_0^0(V) \cong F$.
By standard notation, we set

$$T^k(V) := T_0^k(V) = \otimes^k V$$
$$\text{and } T_k(V) := T_k^0(V) = \otimes^k V^* = T^k(V^*)$$

For the remainder of the article, fix a basis as before, say $\mathcal{B} = \{e_1, \cdots, e_n\}$ of V and let the dual basis to $\mathcal{B}$ be $\mathcal{B}^* = \{\omega_1, \cdots, \omega_n\}$.

Now, observe that $\mathcal{T}_1^1(V) \cong \mathcal{L}(V, V)$ via the following correspondence, a typical element A $\in$ $\mathcal{T}_1^1(V) = V \otimes V^*$ looks like a linear combination of elements the form $v \otimes \omega$ with $v \in V, \omega \in V^*$. So, let $A = v \otimes \omega$, then it defines a linear map $\tilde{A} : V \twoheadrightarrow V$ via $\tilde{A}(u) = \omega(u)v \; \forall \; u \in V$. Conversely, suppose $S \in \mathcal{L}(V, V)$. Then S corresponds to the element

$$S = \sum_{i,j=1}^{n} S_j^i e_i \otimes \omega^j \in \mathcal{T}_1^1(\mathbb{V})$$

where $S_j^i = \omega^i(S(e_j))$.
In typical bra-ket notation, the above would read,

$$S = \sum_{i,j=1}^{n} |e_i\rangle \langle e_i| S |e_j\rangle \langle e_j| = \sum_{i,j=1}^{n} S_j^i |e_i\rangle \langle e_j|$$

where $S_j^i = \langle e_i|S|e_j\rangle$.

Finally, let us make connection with the coordinate based approach used in physics. Let $A \in \mathcal{T}_s^r(V)$. One checks that a basis for $\mathcal{T}_s^r(V)$ is given by

$$\mathcal{B}_s^r = \{e_{i1} \otimes \cdots \otimes e_{ir} \otimes \omega^{j1} \otimes \cdots \otimes \omega^{js} : 1 \leq i_1, \cdots, i_r, j_1, \cdots, j_s \leq n\}$$

With respect to this basis, the components of A are given by

$$A = A^{i_1 \cdots i_r}_{j_1 \cdots j_s} e_{i_1} \otimes \cdots \otimes e_{i_r} \otimes \omega^{j_1} \otimes \cdots \otimes \omega^{j_s}$$

where A is being viewed as an element of Mult(V*, · · · , V*, V, · · · , V; F). Thus, using Einstein notation,

$$A^{i_1 \cdots i_r}_{j_1 \cdots j_s} = A(\omega^{i_1}, \cdots, \omega^{i_r}, e_{j_1}, \cdots, e_{j_s})$$

4. Concluding Remarks

So, now we have a fair bit of idea of what tensors are. However, we have still not shown why in physics, they are described by their transformation properties. The truth is, in physics, we are not dealing with tensors but tensor fields, that is, we assign to each point in spacetime a tensor.

In such a case, since spacetime is modeled by a 4-dimensional smooth manifold *M* with a *pseudo-Riemannian metric* with Lorentz signature (in both, special and general relativity, in the former curvature is 0 while it is not so in the latter), we are dealing with tensor fields on manifolds. For those familiar with basic differential geometry, the vector space V in the above discussion is to be replaced by the tangent space T_pM. In this language, tensor fields correspond to smooth sections of the *tensor bundle* on the manifold *M*. The transformation properties then followfrom the compatibility conditions on the coordinate charts for the tensor bundle. Another important concept is that of *symmetric* and *alternating* tensors, which we have not defined. These are the tensors which behave nicely under action of the elements of the permutation group. In fact, a *metric*(Riemannian or pseudo-Riemannian) on the manifold is a symmetric (0,2) non-degenerate tensor field. Alternating tensors are extremely important and give rise to the notion of *differential forms* which are used to define integration on manifolds.

Also, we can form vector space

$$T(V) := \otimes^{\infty}_{k=0} T^k(V)$$

which is called the tensor algebra and where the multiplication of two tensors of rank (r, s) and (r', s') is given by the outer or *tensor product*, giving a tensor of rank $(r + r', s + s')$. Viewing it as an algebra is extremely useful since it contains as subalgebras the exterior algebra, symmetric algebra, Clifford algebra and Weyl algebra each of which is important in physics. Mathematically, it is important since it is the free algebra on V.

We hope that this article has persuaded the readers to view tensors in a different light and encouraged them to explore some topics in differential geoemetry, one of the foundations of modern physics.

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Message from a different point in SPACE-TIME

- Anchal Gupta
Graduate Student of Applied Physics, Caltech

It is interesting the way your mind grows and perception about the world around you changes. After graduating from IITB, I felt I have learned a lot through my temporal journey of 4 years. But as a new alumnus, I realized there is much more to understand and retrospect once you leave the country, get into a foreign research setting and switch gears into graduate studies. In this article, I would like to share some thoughts I have been having for last one and a half year about how situations, courses, people and research are different and similar between a college in US and IITB. With this, I have some suggestions (or I should say food for thought) for the people who are shaping how physics department at IITB would look in future.

So starting with some good things I miss about our department, it has a very nice open door policy with almost all professors. The atmosphere is quite helpful to students for interacting with professors without much intimidation or writing emails. I feel I had a great time at IITB when I was able to approach any professor with my stupid questions almost within a day or two.

Another important plus point that our department has are open studying spaces, in the computer lab and the library. These places are much more than a bunch of table, chairs, and computers. It provides a conducive environment for group studies and collaborating in work. I have seen the department pushing a lot towards such facilities and we are definitely much better off than other departments on this end.

And then there are some points, towards which, our department is taking steps but is falling behind the optimum level in my opinion. One of them is the role of faculty advisors in our department. Across different batches, I have seen some advisors who get involved a lot in the personal and academic well being of students in their batch, while some think of it as a rubber stamp formality. Looking back, I feel faculty advisors are much more important part of our time in IITB then most people think or feel. Particularly, given that a significant amount of students enter EP not knowing exactly what they are interested in, I feel faculty advisors should be more involved with each student to guide them through their choices of electives and summer projects.

Additionally, our department and IITB as a whole lacks severely in mental health care and in my opinion, students of our department are more prone to depression. I feel advertising the facilities we have, removing the stigma about mental health and having some dedicated people to talk and take care of such students is the need of the hour as a lot of students every year are falling into the vicious cycle of depression and unproductivity with no support.

Another point where our department is better off than others but is not quite there yet is the way assignments and exams are held in our courses. It is great that many of our courses have assignments with a contribution to grades but I feel the contribution is too little. More trust can be put on the students that they will not copy assignments and emphasis can be given on learning by practicing than on learning by preparing for exams. I feel when trust is shown to the students, they reciprocate by taking the responsibility and doing the right thing. While, when skepticism is shown on the morality of students, they tend to develop on the other side, growing into less responsible people.

Such trust can be shown more by making more fraction of exams open book or take home, by providing open access of facilities like printing and scanning without fearing misuse and by having more courses as electives than core.

And the last point takes us to a place where we need much more changes in my opinion. I feel I learned a lot in IITB but the order I took courses in made it difficult to understand a lot of basics. Also, we understood about the different fields of research at different times which inevitably affected our choices of research career a lot. Broadly, the courses should be ordered such that we get an overview of different fields as early as possible in initial semesters followed by higher-level courses in our choice of the field towards the final 3 semesters. If possible, there should be a seminar course for second-year students where they have to attend lectures given by professors, post docs or grad students of the department about their research every week. This seminar course can prove to be very important in choosing the field of research for the students.

In addition, the involvement of students in research can be increased a lot more than it is currently. There are a lot of tasks which undergraduates can do (and actually do for groups in foreign universities in summer projects), which can be taken advantage of during semesters. If each group actively hires or rotates 2-3 undergraduate students every semester, it would improve research output as well as train almost all the students for a research career ahead.

Finally, there are certain points that I feel should be thought about but are not immediately possible. Our department lacks in certain hot fields of research or if it doesn't, at least doesn't have any courses teaching the same. Quantum optics, quantum computation, quantum information, cell biophysics and gravitational wave astronomy (theory and instrumentation both) are few that come to my mind. It would be awesome if our department gets more faculty and introduces courses in these fields.

I understand this article looks more like criticism than praise, but hey, we all want to improve right. I still feel Department of Physics at IITB is really good and I miss it for so many reasons. My points above, reflect my opinion from this space point at Caltech, from this time moment of my second year in PhD. It can be wrong and it can change, but the main point is, we need to do such introspection and retrospection much more than we normally do, when we are at the right place, at the right time.

Do you UNDERSTAND DATA?

- Sukanya Kudva, *3rd Year EP*

This is the era of data science. With the present technological sophistication, people have become capable of collecting and handling large data sets. Think of it- there are probably a ten thousand students in the campus. If you can manage to collect data about how each student distributes his/her time to different tasks throughout the day, you could look for patterns. And if you are crazy enough, pillage the date further to come up with some absurd theories. (Catching a tumtum could be more likely if you take a bath everyday or number of assignments over weekends may be positively correlated with how averse you are to mess food). Anyways, if you were to come to more 'scientific' conclusion, you would require more carefully planning with multiple experiments, large number of sample observations and sophisticated analysis. But after meticulous efforts in acquisition and analysis, are the conclusions always correct? And if they are, is it always easy to interpret them correctly without being misled?

Deceptively Presented

In 2011, there was a general buzz in the media about the discovery of a new particle in Collider Detector at Fermilab (CDF)[1]. One news article on the subject read, "there is a 99.7% chance of the result being correct". Wait, how did they come up with this figure? When experimental attempts are made to verify a theory, the general procedure is to define a null hypothesis. The natural speculation that there is no relation whatsoever between the observed quantities is codified in this conjecture. And the aim then, is to reject the null hypothesis with supporting data.

In this case, the null hypothesis presumed that no such new particle existed. Based on this hypothesis, the chance of an extreme event such as that observed at CDF was reported to be 0.3% if it were only a statistical fluctuation indicative of no new Physics involved. But does that translate to the probability of the result itself being true as 99.7%?

source: xkcd.com

Say your friend is a coin collector and an American. You come up with a distribution that predicts the probability that s/he has a certain number of obsolete Indian coins. Your theory deems it highly unlikely that an American has more than 10 such coins (say, a probability of 0.08).

But now you are surprised to know that that s/he has plenty of these fabled coins (s/he even sends photographs of it as proof). What do you conclude? Does your friend have an 8 percent chance of being American? May be not. This is slightly confusing but if you do take a while to reexamine the scenario, you would agree that it's an absurd conclusion to make. You can't conclude the probability of a theory being correct based on how well it explains your data. You must compare it with other theories to reach that conclusion. And therein lies the loophole. No wonder the CDF's experimental results were treated with skepticism by several people in the scientific community.

Systematic ordeal

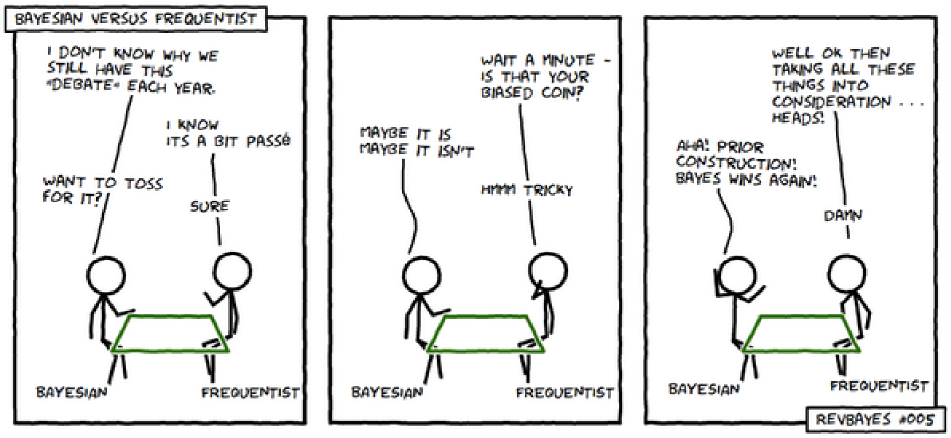
When carrying out an experiment, diligent endeavours are made to identify and minimise the possible sources of systematic errors - unavoidable errors inherent in the instrumentation. Despite such fastidiousness, these errors still creep up in from the most unexpected of places.

Metrologists have been in pursuit of a more accurate Newton's Gravitational constant 'G' for quite a long time. When Henry Cavendish measured this value, the error bar in his pioneering contribution was estimated to be around 1%. And today, the accuracy of most experiments is still only about 50 parts per million. With gravity being the most feeble of the fundamental forces and hence the consequent experimental limitations, 'G' has a comparatively larger uncertainty than most other physical constants.[2]
In 1996, a team from Braunschweig, Germany devised a novel method, different from the traditional or modified Cavendish setup to measure G. The simple idea was to let test masses, afloat in a mercury bath, interact with source masses to generate torque. This torque was to be subsequently nullified by an electrostatic torque exerted by electrodes placed near the test masses. Unfortunately for the team, what was supposed to be an experimental tour de force turned out to be a 8-year long pursuit to reason out an anomaly. The team determined a gravitational constant which was 50 standard deviations away from the then accepted value of G. 50 standard deviations! It is a nightmare for any experimentalist.

And it was perhaps even worse to realise that the culprit was a neglected calibration term all along. In a 2004 paper published by the team, neglecting the cross capacitance term in their electrostatic unit was identified as the flaw.

Background check?

One obvious challenge in analysing data is the subjectivity of approaches inherent in the theory. Primarily, there are two main statistical approaches: the Bayesian approach and the Frequentist



source:https:/twitter.com/revbayes/status/506577193804111872

approach. A Bayesianist is naturally more inquisitive; s/he would do a background check of where different hypotheses stand relatively before the experiment and account for it. A Frequentist would simply not care; s/he would prefer to start afresh.

In the recent past, there have been many attempts to estimate the mass of a neutrino, a particle which was previously attributed to be massless. However, most of the experiments so far have got an imaginary (unphysical) estimate for the mass. Physicists are trying to find a statistical bound for the true mass. Therefore, the reasonable question of which statistical approach to adopt becomes fundamental. Frequentists will want to calculate something called a 'confidence interval' while Bayesianists would rely on a 'credible interval' and we need to understand both to know what our decision entails.

A hypothetical case could illustrate the underlying basic difference in their philosophies.[3]
Say your friends challenge you over a game - an extension to the Christmas Secret Santa. One of them anonymously leaves a gift for you choosing from your wishlist. You are to guess who it was. If you adopt the Frequentist approach, you would make a list of gifts for each friend that s/he is mostly likely to give you. So then, every gift 'could' fall into the 'most likely gift list' of some of your friends. This is the confidence interval of every gift. A Bayesian could still quibble about this method - they would point out that if none of your friends are likely to buy you some present, you wouldn't have a 'confidence interval' corresponding to it at all! So then, they propose another method. Starting with the prior probabilities - a probability distribution of their wise choice - that some friend volunteers to give a gift, they construct a 'credible interval' for every gift based on which of them are most likely to give a gift. But this also has its own issues. What if a particular friend isn't included in any of the credible intervals? And how rational is the method by which they come up with prior probabilities?

The arguments on both sides could go on forever. There is no right or wrong method. But the differences in these two statistical constructs is something that we could always appreciate.
So, the next time you see a newspaper headline saying- 'the bound to the value is estimated to be..' or '..very low odds of this being a statistical fluctuation' or '.. highly confident about a genuine discovery', read beyond the numbers. Try to see how the numbers were calculated. And if you ever end up with absurd values in an experiment, consider for a moment that you might have overseen a few systematic errors.

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